NUMBER SENSE

8 TH GRADE

Perfect Squares Tiles Activity

1. Using the square tiles, make the smallest perfect square you can.
    a. How many tiles did you use?
    b. What are the dimensions of your square (length and width)?
2. Using more tiles, make the next smallest perfect square you can.
    a. How many tiles did you use?
    b. What are the dimensions of your square (length and width)?
3. Make the next smallest perfect square you can.
    a. How many tiles did you use?
    b. What are the dimensions of your square (length and width)?
4. Make the next smallest perfect square you can.
    a. How many tiles did you use?
    b. What are the dimensions of your square (length and width)?

5. Using all your given tiles, make the biggest perfect square you can.

    a. How many tiles did you use?

    b. What are the dimensions of your square (length and width)?

6. What does it mean to square a number?

7. What does it mean for a number to be a perfect square? Can just any number be considered a perfect square, why or why not?

8. What does it mean to take the square root of a number? Think back to your tiled squares, what part of the diagram represents the square root?

9. Complete the table below by listing all the perfect squares you discovered on the front side in order least to greatest.

A Number that is a Perfect Square	Dimensions of the Square (length x width)	What is the Square Root of the Perfect Square Number?
Example: 1		

10. What is the algebraic relationship between squaring a number and taking the square root of a number?

11. Complete the following table without a calculator – estimate the solutions as best you can.

Solution to the expression x^2	Dimensions of a tiled square	Square Root of Each Solution
Example: 4	2 x 2	2
3		
6		
12		
20		
34		
46		
57		
72		

Explain how you chose numbers to complete the table above.

12. Use tables on the next 2 pages. Analyze each table and explain what the students were thinking when they completed the table and if you agree with their method. Choose a table that you feel is the most accurate.

    a. Table A

    b. Table B

    c. Table C

Table A		
Solution to the expression x^2	Dimensions of a tiled square	Square Root of Each Solution
Example: 4	2 x 2	2
3	1.5 x 1.5	1.5
6	3 x 3	3
12	6 x 6	6
20	10 x 10	10
34	17 x 17	17
46	23 x 23	23
57	28.5 X 28.5	28.5
72	36 x 36	36

Table B		
Solution to the expression x^2	Dimensions of a tiled square	Square Root of Each Solution
Example: 4	2 x 2	2
3	3 x 1	2
6	3 x 2	2.5
12	6 x 2	4
20	2 x 10	6
34	17 x 2	9.5
46	23 x 2	12.5
57	28.5 X 2	15.25
72	9 x 8	8.5

Table C		
Solution to the expression x^2	Dimensions of a tiled square	Square Root of Each Solution
Example: 4	2 x 2	2
3	1.8 x 1.8	1.8
6	2.4 x 2.4	2.4
12	3.3 X 3.3	3.3
20	4.5 x 4.5	4.5
34	5.8 x 5.8	5.8
46	6.7 x 6.7	6.7
57	7.5 x 7.5	7.5
72	8.4 x 8.4	8.4

Squares and Square Roots Practice

Simplify each expression by using the exponent. Do not use a calculator.

1. 4^2
2. 8^2
3. 5^2
4. 12^2
5. 3^2
6. 9^2
7. 10^2
8. 5^2
9. 7^2
10. 11^2
11. 1^2
12. 13^2
13. 15^2
14. 6^2
15. 20^2

Without using a calculator, find the square root of each of the following numbers. For any non-perfect squares, estimate the square root to the nearest tenth.

1. $\sqrt{0}$
2. $\sqrt{144}$
3. $\sqrt{33}$
4. $\sqrt{49}$
5. $\sqrt{81}$
6. $\sqrt{25}$
7. $\sqrt{121}$
8. $\sqrt{44}$
9. $\sqrt{8}$
10. $\sqrt{49}$
11. $\sqrt{56}$
12. $\sqrt{2}$
13. $\sqrt{9}$
14. $\sqrt{20}$
15. $\sqrt{90}$

Perfect Cubes Activity

Using your perfect square tiles and perfect cube cubes, create each of the following models:

Length of each side (Dimensions)	Sketch of your perfect square with dimensions labeled.	Number of Tiles used in Creating a Perfect Square	Sketch of your perfect cube with dimensions labeled.	Number of Cubes used in Creating a Perfect Cube
1				
2		Ex. 4	8	
3				

Length of each side (Dimensions)	Sketch of your perfect square with dimensions labeled.	Number of Tiles used in Creating a Perfect Square	Sketch of your perfect cube with dimensions labeled.	Number of Cubes used in Creating a Perfect Cube
4				
5				
6				
7				

1. Apply what you already know about perfect squares to answer the following questions.

   a. What does it mean to cube a number? Explain using full sentences.

   b. Which part of your model represents the value of cubing a number, ex. the value of 4^3 is represented where, how? Explain using full sentences.

   c. Which part of your model represents the cube root of a number, such as the cube root of 8 written as $\sqrt[3]{8}$? Explain using full sentences.

   d. What does it mean for a number to be a Perfect Cube? Would you say that all numbers are perfect cubes, explain your answer?

e. Estimate the value of the following expressions without using a calculator.

$\sqrt[3]{21}$ $\sqrt[3]{900}$ $\sqrt[3]{100}$ $\sqrt[3]{12}$ $\sqrt[3]{320}$

Extension Activity:

Squares and Square Roots, Cubes and Cube Roots are applied in geometry. Let's think about how squares and cubes relate to geometry.

1. Match the following geometry vocabulary to the correct part of your models created and sketched in the activity today.

   a. Perimeter_____ d. Number of cubes inside a perfect cube

   b. Area _____ e. Sum of side lengths on a perfect square

   c. Volume _____ f. Number of tiles inside a perfect square

2. Application questions using perimeter, area, and volume. You might find it helpful to draw a picture.

   a. The area of a square is 36 in^2. What is the perimeter of the square?

   b. The volume of a cube is 27 ft^3. What is the area of the square base (flat square on the bottom side) of the cube?

   c. The perimeter of a square is 28 cm, what is the area of the same square?

   d. The area of a square is 81 m^2. What are the dimensions of this square?

e. The volume of a cube is 125 ft^3. What are the dimensions of the cube?

3. Complete the following table and then use this table to create the following graphs.

Perimeter of Perfect Squares		Area of Perfect Squares		Volume of Perfect Cubes	
Length of side (cm)	Perimeter (cm)	Length of side (cm)	Area (cm^2)	Length of side (cm)	Volume (cm^3)
1		1		1	
2		2		2	
3		3		3	
4		4		4	
5		5		5	

Perimeter Graph

Area Graph

Volume Graph

4. Look at your graphs and explain the pattern you see in each graph. How are the graphs different and what is causing this changes in pattern? Discuss these questions in your group and write your group thoughts.

   a. Perimeter Graph

   b. Area Graph

   c. Volume Graph

   d. Differences in graphs

Real Number Venn Activity Sheet

1. Give an example of a repeating decimal and a terminating decimal.

2. If the number 0.77 is displayed on a calculator that can only display ten digits, can we consider this an infinitely expanding decimal? Explain:

3. Which of the following numbers have a repeating pattern of digits? Explain each answer

a) 7.71711711171111711111….

b) 0.34

c) 0.1234567…

d) 0.538461538461…

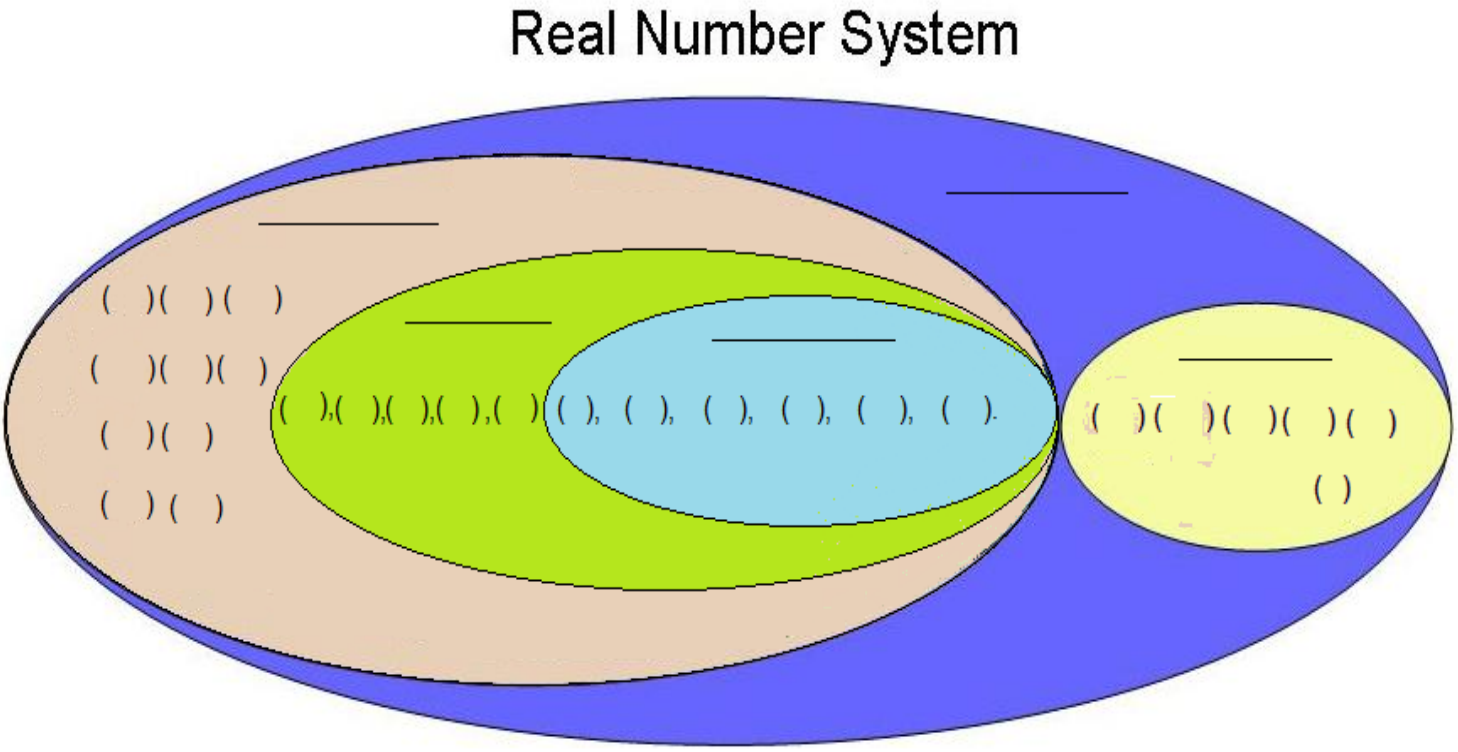
4. True or False (If false, give an example to justify)

a. All whole numbers are integers.

b. All integers are whole numbers

c. All rational numbers are integers

d. All integers are rational



Indicate whether the numbers are rational or irrational and why.

1. $\sqrt{25}$

2. $\sqrt{24}$

3. $-\sqrt{35}$

4. 3/8

5. 1/7

6. 0.222…

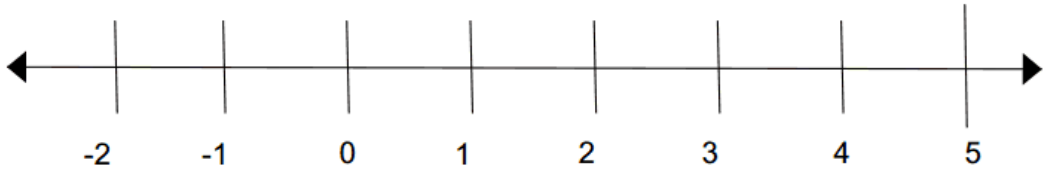
7. 11/24

8. 1.414213

9. $\sqrt{2} = 1.414213\ldots$

10. Order the following rational and irrational numbers on the number line

$\sqrt{12}$ $\sqrt{9}$ $-1\frac{1}{4}$ -2 $\frac{5}{8}$



Exponents Investigation - Multiplication

Simplify the following expression by expanding the expression into what it means for multiplication:

Expression with Exponents	Expression Expanded into Multiplication	Answer to Simplified Expression (answer after multiplied)	Rewritten Expression Using only one Exponent
$2^2 \cdot 2^4$	2·2·2·2·2·2		
$3^3 \cdot 3^4$			
$5^2 \cdot 5^2$			
$3^3 \cdot 4^2$			
$x^4 \cdot x^6$		Can you multiply when you don't know the value of x? Answer this below.	
$z^7 \cdot z^8$			
$c^5 \cdot c^{15}$			
$d^{-5} \cdot d^8$			
$2^2 \cdot s^2 \cdot 3^2 \cdot f^2$			

Can you multiply when you don't know the value of x?

Write a rule to summarize the property of multiplication with exponents.

Exponents Investigation Two - Division

Simplify the following expression by expanding the expression into what it means for multiplication:

Expression with Exponents	Expression Expanded into Multiplication	Answer to Simplified Expression (answer after multiplied)	Rewritten Expression Using only one Exponent
$\frac{2^4}{2^2}$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2}$	4	
$\frac{3^4}{3^3}$			
$\frac{5^2}{5^2}$			
$\frac{3^3}{4^2}$			
$\frac{x^6}{x^4}$		Can you divide when you don't know the value of x? Answer this below.	
$\frac{z^8}{z^7}$			
$\frac{c^{15}}{c^5}$			
$\frac{d^{15}}{d^8}$			
$\frac{2^5s^7}{2^2s^2}$			

Can you multiply when you don't know the value of x? Can you simplify when you don't know the value of x?

Write a rule to summarize the property of exponents when division is the operation.

Exponents Investigation Three – A Power Raised to a Power

Simplify the following expression by expanding the expression into what it means for multiplication:

Expression with Exponents	Expression Expanded into Multiplication	Answer to Simplified Expression (answer after multiplied)	Rewritten Expression Using only one Exponent
$(2^4)^2$	(2·2·2·2)(2·2·2·2)		
$(3^4)^3$			
$(5^2)^4$			
$(3^3)^1$			
$(x^6)^4$			
$(z^8)^7$			
$(c^{15})^5$			
$(d^{15})^8$			
$(2^3sm^7)^2$			

Can you multiply when you don't know the value of x? Can you generate an equivalent expression when using variables?

Write a rule to summarize the property of simplifying a power raised to another power.

Negative and Zero Exponents

1. Complete the following tables using the exponents to generate equivalent values.

Expression	Equivalent Value Simplified
2^7	
2^6	
2^5	
2^4	
2^3	
2^2	
2^1	

Pattern?

Expression	Equivalent Value Simplified
5^7	
5^6	
5^5	
5^4	
5^3	
5^2	
5^1	

Pattern?

Expression	Equivalent Value Simplified
3^7	
3^6	
3^5	
3^4	
3^3	
3^2	
3^1	

Pattern?

Expression	Equivalent Value Simplified
10^7	
10^6	
10^5	
10^4	
10^3	
10^2	
10^1	

Pattern?

2. Look back at your tables in question one. Look at the equivalent values column and look up the column – what pattern do you notice for each table as the exponent increases by one each row? Write down the pattern for each table.

3. Compare your answer to question 2 with a partner.

4. How can you use the pattern you just found to complete the following tables by going down each table – decreasing your exponent by one each row?

5. Compare your answer to question 4 with a partner.

Expression	Equivalent Value Simplified
2^2	
2^1	
2^0	
2^{-1}	
2^{-2}	
2^{-3}	
2^{-4}	

Pattern?

Expression	Equivalent Value Simplified
5^2	
5^1	
5^0	
5^{-1}	
5^{-2}	
5^{-3}	
5^{-4}	

Pattern?

Expression	Equivalent Value Simplified
3^2	
3^1	
3^0	
3^{-1}	
3^{-2}	
3^{-3}	
3^{-4}	

Pattern?

Expression	Equivalent Value Simplified
10^2	
10^1	
10^0	
10^{-1}	
10^{-2}	
10^{-3}	
10^{-4}	

Pattern?

6. Look at all four tables, what is the value of 2^0, 3^0, 5^0, and 10^0. What does this tell you about x^0?

7. Compare your answer to question 6 with a partner.

8. Looking back at your tables is 3^{-2} equivalent to 3(-2)? Do the rules of multiplication with a negative number apply when the negative number is an exponent? (Write your answers here)

9. Compare your answer to question 8 with a partner.

10. What is the rule to summarize the property of any number to the 0 power?

11. What is the rule to summarize the property of negative exponents? (What effect does the negative exponent have overall?)

Writing Numbers in Scientific Notation

1. Complete the following tables using a calculator – be looking for a pattern as you go that will help you to answer questions a and b.

   a. What happens when you multiply by 10 to a positive power?

   b. What happens when you multiply by 10 to a negative power?

Expression	Simplified Equivalent Value
6×10	
4×10^2	
2×10^3	
6.45×10	
4.32×10^2	
2.678×10^3	
8.23491×10^5	

Expression	Simplified Equivalent Value
6×10^{-1}	
4×10^{-2}	
2×10^{-3}	
6.45×10^{-1}	
4.32×10^{-2}	
2.678×10^{-3}	
8.23491×10^{-5}	

2. Without using a calculator, convert the following numbers from scientific notation into standard decimal notation using your patterns from the tables.

6.17×10^{3} 7×10^{4} 7.31×10^{6}

5.4×10^{-8} 6.7×10^{-3} 9.59×10^{2}

3. Notes for Proper Scientific Notation:

Multiplying in Scientific Notation

1. What is the associative property of multiplication, for example 3x(6x)?

2. Multiply the following terms to generate equivalent expressions.
   a. $5x(7x^2)$ b. $10z^3(8z^4x^7)$ c. $2(3d)^2$

3. How can the associative property be applied to the following problem to generate a simpler equivalent expression?
   $(1.346 \times 10^3)(1.2 \times 10^5)$

4. Apply the associative property to generate an equivalent expression:
   $(2.764 \times 10^5)(5.421 \times 10^6)$

5. If your equivalent expression to problem 4 is not written in scientific notation, convert it to proper scientific notation now.

6. Generate an equivalent expression written in scientific notation for the following:

$(4.56 \times 10^{-5})(7.93 \times 10^{3})$

7. Describe in full sentences how to multiply two numbers that are written in scientific notation.

8. Generate equivalent expressions written in scientific notation for the following:
   a. $(3.45 \times 10^{7})(5.67 \times 10^{4})$

   b. $(1.42 \times 10^{-2})(5.26 \times 10^{6})$

   c. $(4.61 \times 10^{8})^{2}$

Dividing in Scientific Notation

1. Generate a simpler equivalent expression for each of the ones below:

   a. $\frac{10z^2x^4}{2z^5x^2}$ b. $\frac{4g^2h^7m^0}{16g^4h^3}$ c. $\frac{8c^5m^{-3}}{12c^5m^2}$

2. Explain your strategy(s)for generating equivalent expressions in question 1.

3. Apply these strategies to generate a simpler equivalent expression, written in scientific notation, for the following:

   $$\frac{5.68 \times 10^4}{4.23 \times 10^2}$$

4. Explain your strategy(s) for generating equivalent expressions when the numerator and denominator are both written in scientific notation.

5. Apply your strategy to generate equivalent expressions for each of the following:

a. $\frac{6.78 \times 10^5}{3.24 \times 10^2}$

b. $\frac{3.465 \times 10^3}{1.68 \times 10^8}$

c. $\frac{6.58 \times 10^{-7}}{2.42 \times 10^4}$

d. $\frac{1.2 \times 10^4}{4}$

Comparing Computer Bytes[1]

Memory Name	Number of Bytes
KiloByte (KB)	1.024×10^3
MegaByte (MB)	1.048576×10^6
GigaByte (GB)	1.073741824×10^9
TeraByte (TB)	$1.099511627776 \times 10^{12}$
PetaByte (PB)	$1.125899906842624 \times 10^{15}$
ExaByte (EB)	$1.152921504606846976 \times 10^{18}$
ZettaByte (ZB)	$1.180591620717411303424 \times 10^{21}$
YottaByte (YB)	$1.208925819614629174706176 \times 10^{24}$

Computer memory is discussed as number of Bytes of available memory space. The more Bytes a computer has, the more data it can store. Use the table above to answer the following questions about computer storage.

1. One Byte contains 8 bits of data. How many bits of data are contained in one KiloByte? (Leave your answer in scientific notation and show all your work)

2. How many times larger is a GigaByte than a KiloByte? Show all your work.

3. How many Bytes are in a 2 GB flash drive? (Leave your answer in scientific notation and show all your work)

[1] ***Web resources used to create this lesson:***
http://en.wikipedia.org/wiki/File_size
http://h10025.www1.hp.com/ewfrf/wc/document?cc=us&lc=en&dlc=en&docname=c00877010,
http://inventors.about.com/od/rstartinventions/a/Ram.htm
http://www.computerhistory.org/timeline/?category=cmptr
http://store.apple.com/us/buy/home/shop_iphone/family/iphone5?product=ME488LL/A&#storage

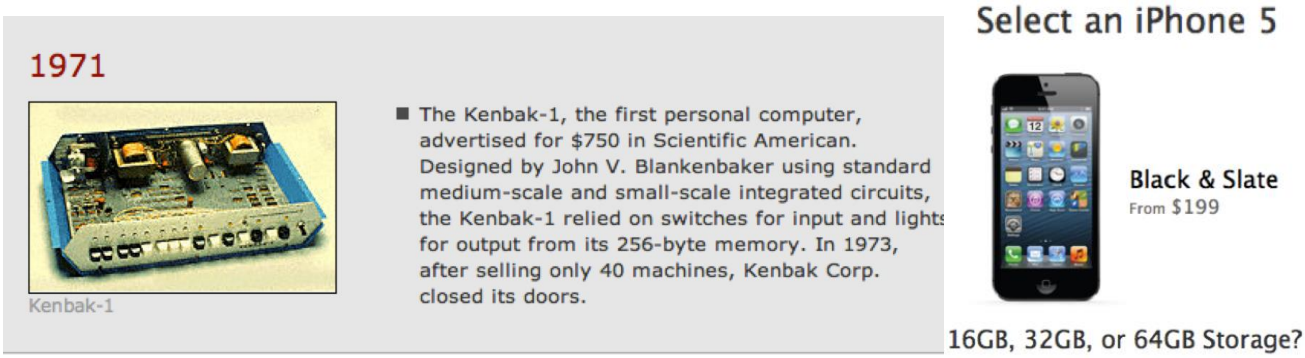
4. The 2013 MacBook Pro laptop can be purchased with either a 500 MB hard drive or a 750 MB hard drive. How many extra Bytes of memory do you have if you purchase the 750 GB over the 500 GB? (Leave your answer in scientific notation and show all your work)

5. Personal computers and external hard drives are now beginning to contain 1 TB of memory space. In terms of Bytes of memory space, how much of an upgrade would it be to order a Mac Book Pro with 1 TB of memory instead of 500 MB? (Leave your answer in scientific notation and show all your work)

6. What percent increase would it be to upgrade your 500 MB notebook to a 1 TB memory? Show all your work.

7. In 1966 Hewitt-Packard released the HP2116A real-time computer with 8KB of memory. In 2013, Hewitt Packard released the HP Pavilion Media Center a6010n, which has 1 GB of memory. How much of an upgrade is this over the past 47 years (By what percent has the memory increased in 47 years of improvement)? Show all your work.

8. Compare the following images:



a. If you purchase the iPhone 5 with 16 GB of memory, it will cost you $199 as of July 9, 2013. How many more bytes of memory space does the iPhone have now than the first personal computer had back in 1971? Leave your answer in scientific notation.

b. What is the percent increase in memory storage from 1971 to 2013? Show all your work.

c. The cost per byte of memory space has become cheaper over time as more efficient methods of storage have been developed, such as the very small Pentium chip. What is the percent decrease in the cost per byte of memory space from 1971 to 2013?